ECONOMIC ORDER QUANTITY (EOQ) – An Extension

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Abstract: *The aim of this paper is to illustrate the determination of the Economic Order Quantity (EOQ) or Economic Number of Orders (ENO) when the Total Ordering Cost (TOC) and Total Handling Cost (THC) are not equally the same. For this purpose, two assumptions of the basic EOQ model - (a) the constant unit handling cost, and (b) the constant per order cost - are relaxed; and instead, their varying conditions are considered. Hence, this paper presents two exhibits to make the readers understand the EOQ at the condition where the Total Handling Cost is not equal to the Total Ordering Cost (THC ≠ TOC). Accordingly, Exhibit 1 relaxes the assumption of 'constant per order cost' and considers the concepts of fixed cost and variable cost for placing an order. Also, Exhibit 2 relaxes the assumption of 'constant unit handling cost' and illustrates with the varying handling cost per unit for the ordering quantity (Q) in an order. These exhibits support the situations where the EOQ (or ENO) can be determined at the condition where THC ≠ TOC. Notably, for both exhibits, appropriate tables and figures are also presented for the readers' convenience to understand the presentation of the paper.*

Keywords: Economic Order Quantity, EOQ, Economic Number of Orders, ENO, incremental cost, ordering cost, handling cost, fixed cost, variable cost

JEL code: C00, C02, C6, C60, C61

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1. INTRODUCTION

A basic deterministic ***Economic Order Quantity*** (***EOQ***) model emphasises that the EOQ can be determined where the Total Handling Cost (THC) equals Total Ordering Cost (TOC). In the model, the Total Incremental Cost (TIC) is represented with the sum of both THC and TOC (TIC = THC + TOC). Hence, the simple EOQ model represents determining the EOQ as a trade off between THC and TOC and at the minimum TIC. In other term, EOQ can be determined when THC = TOC, provided that certain assumptions in place.[1]

Practically, when the assumptions of deterministic EOQ model are relaxed with changes, it is possible to explore that determining EOQ cannot be with the condition THC = TOC. This paper aims to illustrate the situations where the EOQ cannot be determined at the point where THC = TOC. In this context, this paper identifies two specific situations to illustrate the same. They are: (a) Unit ordering cost varies with the ***number of orders to be placed*** annually, and (b) Unit holding cost varies with the ***size of an order*** to be placed. It is notable that the assumptions of simple deterministic EOQ model, specifically the constant unit order cost and the constant unit inventory handling cost, are relaxed and considered varying.

[1] The determination of EOQ consists of the following assumptions: the EOQ will be determined for every product individually in a business, annual requirement (Demand) for product in units is known with certainty, Ordering cost is known and constant throughout the year, Inventory handling cost is known and constant throughout the year, No cash or quantity discount is allowed, The ordered quantity of the product is delivered at once as a single batch, Immediate replenishment of ordered quantity on time and no delay and stock shortage, and Constant lead time is only allowed and no fluctuation is permitted (Senthilnathan, 2019; source: http://ssrn.com/abstract=3475239).

To explore the EOQ determination where TOC ≠ THC, the rest of this paper is organised with the EOQ with the relaxed assumptions (includes Exhibit 1 and Exhibit 2) and a concluding remarks to make the readers understand the aim of this paper clearly.

2. ECONOMIC ORDER QUANTITY WITH TWO THE RELAXED ASSUMPTIONS

Though there are a number of assumptions taking place to explain the simple deterministic EOQ model, this paper relaxes specifically two of them to explain the situation where the EOQ can also be determined with the condition of TOC ≠ THC. In this context, this paper demonstrate that the unit ordering cost varies with the ***number of annual orders to be placed***, and the unit holding cost varies with the ***ordering size of a product*** to maintain in the warehouse.

To make the readers to clearly understand the situations, the exhibits are relatively presented with examples.

Exhibit 1: Unit Ordering Cost varies with the number of annual orders

In this exhibit, the assumption "***constant ordering cost per order***" of primary EOQ model is relaxed with that per order cost is subject to number of annual orders to be placed. Accordingly, consider the following information to understand the relaxed assumption of per order cost.

ABC Ltd. has demand for its material ***D =*** 1,600 units of tons that accounts for 200 working days at the usage rate of 8 units of tons per day. In this context, the unit handling cost (C_H) is \$ 9.00, (i.e., $\boldsymbol{C_H} = 9$). However, for placing orders, there is an annual fixed cost of \$ 400.00 plus the variable cost of \$ 18.00 per order.

It is notable that determining EOQ is almost similar to determine Economic Number of Orders (ENO), where $\boldsymbol{EOQ} = \frac{\boldsymbol{Demand\ (D)}}{\boldsymbol{Number\ of\ Orders\ (N{=}ENO)}}$ and $\boldsymbol{ENO} = \frac{\boldsymbol{Demand\ (D)}}{\boldsymbol{Ordering\ Quantity\ per\ order\ (Q{=}EOQ)}}$. Explicitly, It is obvious that determining EOQ or ENO is complementary to each other. In this example, the explanation takes place in determining the ENO, since the per-order cost depends on annual number of orders.

According to the information available, per-order cost should be determined with the annual fixed cost (\$ 400) plus the variable cost (\$ 18). Accordingly, per-order cost (C_O) can be determined as:

$$C_O = \frac{400}{N} + 18$$

Where C_O = per-order cost and N = annual number of orders.

It is possible to explain that the per-order cost is the sum of the average fixed cost $\left(\frac{400}{N}\right)$ plus variable cost (18).

Therefore, annual Total Ordering Cost (TOC) $= N.C_O = N\left(\frac{400}{N} + 18\right) = 400 + 18N$

Alternatively, the TOC can be determined with the annual fixed ordering cost ($ 400) plus the variable cost ($ 18) per order. As we assume that N = annual number of orders, the annual TOC can also be given as:

TOC = Annual fixed cost (= 400) + Annual variable cost of orders (= 18 N)

TOC = 400 + 18 N --- (1)

In terms of Quantity (Q), $TOC = 400 + 18\left(\frac{D}{Q}\right)$ --- (2)

Further, it is important to determine the Total (annual) Handling Cost (THC) for the materials.

Thus, THC = Average stock in the warehouse $\left(\frac{Q}{2}\right)$ * Unit handling Cost (C_H)

$THC = \left(\frac{Q}{2}\right).C_H$ --- (3)

As we focus on determining ***ENO***, it is important to transform the above ***Q*** into ***N***.

As we know that $Q = \frac{Demand\ (D)}{Number\ of\ Orders\ (N)} = \frac{D}{N}$, substituting ***Q*** in terms of ***N*** can result in

$$THC = \left(\frac{D}{2N}\right).C_H = \frac{D.C_H}{2N}$$

As ***D*** = 1,600 units of tons and C_H = 9, substituting these values in the above THC will give

$THC = \frac{1600*9}{2N} = \frac{7200}{N}$ --- (4)

Now, the Total Incremental Cost TIC = TOC + THC --- (5)

$$TIC = (400 + 18N) + \left(\frac{7200}{N}\right)$$

As we need to get the Economic (optimum) Number of Orders (***ENO***), it is necessary to differentiate the above ***TIC*** with respect to ***N***.

This can result in as:

$$\frac{d(TIC)}{dN} = (18) + \left(\frac{-7200}{N^2}\right)$$

For maximum or minimum of TIC for a optimum value of ***N***, $\frac{d(TIC)}{dN} = 0$. Therefore,

$$\frac{d(TIC)}{dN} = (18) + \left(\frac{-7200}{N^2}\right) = 0$$

This can result in $18\boldsymbol{N2} = 7200$ and $N^2 = \frac{7200}{18} = 400$

Hence $ENO = N^* = \sqrt{400} = 20$ and

$EOQ = Q^* = \left(\frac{D}{N^*}\right) = \frac{1600}{20} = 80$

From the above results, it is notable that with respect to ENO and EOQ, the TOC and THC are different and not the same/equal at the respective optimal point (see Table 1).

Table 1: TOC, THC and TIC with respect to ENO and EOQ

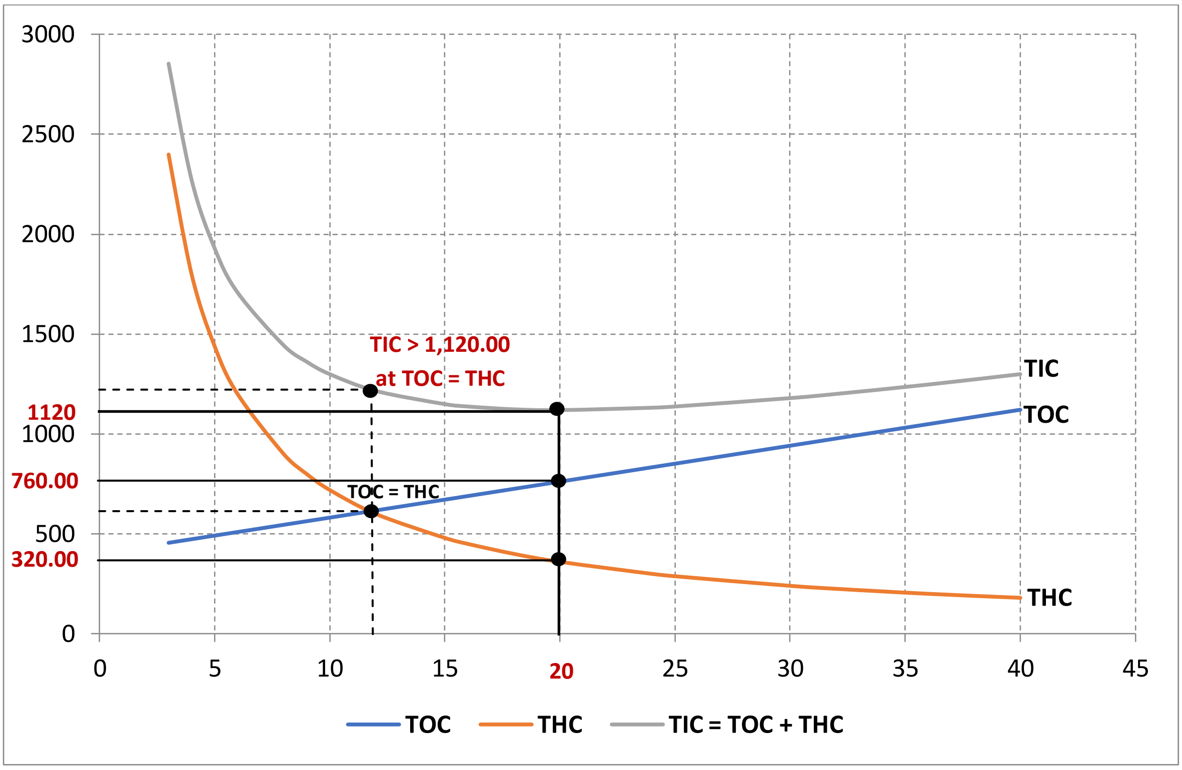
With respect to	Total Ordering Cost (TOC)	Total Handling Cost (THC)	TIC = TOC + THC
ENO = N*	From Equation 1: $TOC = 400 + 18N^*$ **TOC** = 400 + 18(20) = **760**	From Equation 4: $THC = \frac{7200}{N^*} = \frac{7200}{20}$ **THC = 360**	From Equation 5: **= 760 + 360** **= 1120**
EOQ = Q*	From Equation 2: $TOC = 400 + 18\left(\frac{D}{Q^*}\right)$ $TOC = 400 + 18\left(\frac{1600}{80}\right)$ $\boldsymbol{TOC = 760}$	From Equation 3: $THC = \left(\frac{Q^*}{2}\right) * C_H$ $THC = \left(\frac{80}{2}\right) * 9$ $\boldsymbol{THC = 360}$	From Equation 5: **= 760 + 360** **= 1120**

Considering the above calculations and information, it is possible to show them in diagrams with respect to ENO and EOQ (see Table 2 to Figure 1 and Table 3 to Figure 2, respectively).

Table 2: TOC, THC and TOC with respect to number of orders to be placed

Number of Orders	Total Ordering Cost (TOC) $\boldsymbol{TOC = 400 + 18N}$	Total Handling Cost (THC) $\boldsymbol{THC = \frac{7200}{N}}$	TIC = TOC + THC
3	454.00	2400.00	2854.00
4	472.00	1800.00	2272.00
5	490.00	1440.00	1930.00
6	508.00	1200.00	1708.00
8	544.00	900.00	1444.00
9	562.00	800.00	1362.00
10	580.00	720.00	1300.00
12	616.00	600.00	1216.00
15	670.00	480.00	1150.00
16	688.00	450.00	1138.00
18	724.00	400.00	1124.00
20	760.00	360.00	1120.00
24	832.00	300.00	1132.00
25	850.00	288.00	1138.00
30	940.00	240.00	1180.00
32	976.00	225.00	1201.00
36	1048.00	200.00	1248.00
40	1120.00	180.00	1300.00

Figure 1: TOC, THC and TOC with respect to number of orders to be placed



Where TOC = Total Ordering Cost, THC = Total Handling Cost, TIC = Total Incremental Cost, and X-axis refers to the number of orders (*N*) to be placed.

In the above diagram, it is notable that the minimum TIC is not reached at the (intersecting) point of TOC and THC, where the TOC = THC.

However, it is notable that ***TOC*** and ***THC*** becomes equal (***TOC = THC***), when about 11.7681 number of orders are placed that can result in ***TOC = THC*** = 611.8252 and ***TIC*** = 1,223.65. The above results can be obtained by equating the both ***TOC*** and ***THC*** and solving for ***N***.

Accordingly, $\boldsymbol{TOC = 400 + 18N}$ and $\boldsymbol{THC = \frac{7200}{N}}$

By equating TOC and THC, $\boldsymbol{400 + 18N = \frac{7200}{N}}$

Hence, $\boldsymbol{18N^2 + 400N - 7200 = 0}$ → $\boldsymbol{9N^2 + 200N - 3600 = 0}$

Solving for ***N*** can result in

$$\boldsymbol{N = \frac{-200 \pm \sqrt{200^2 - 4 * 9 * (-3600)}}{2 * 9}}$$

$\boldsymbol{N = \frac{-200 \pm 411.8252}{18}}$ and solving for positive, $\boldsymbol{N = \frac{-200 + 411.8252}{18} = \frac{211.8252}{18} = 11.7681}$

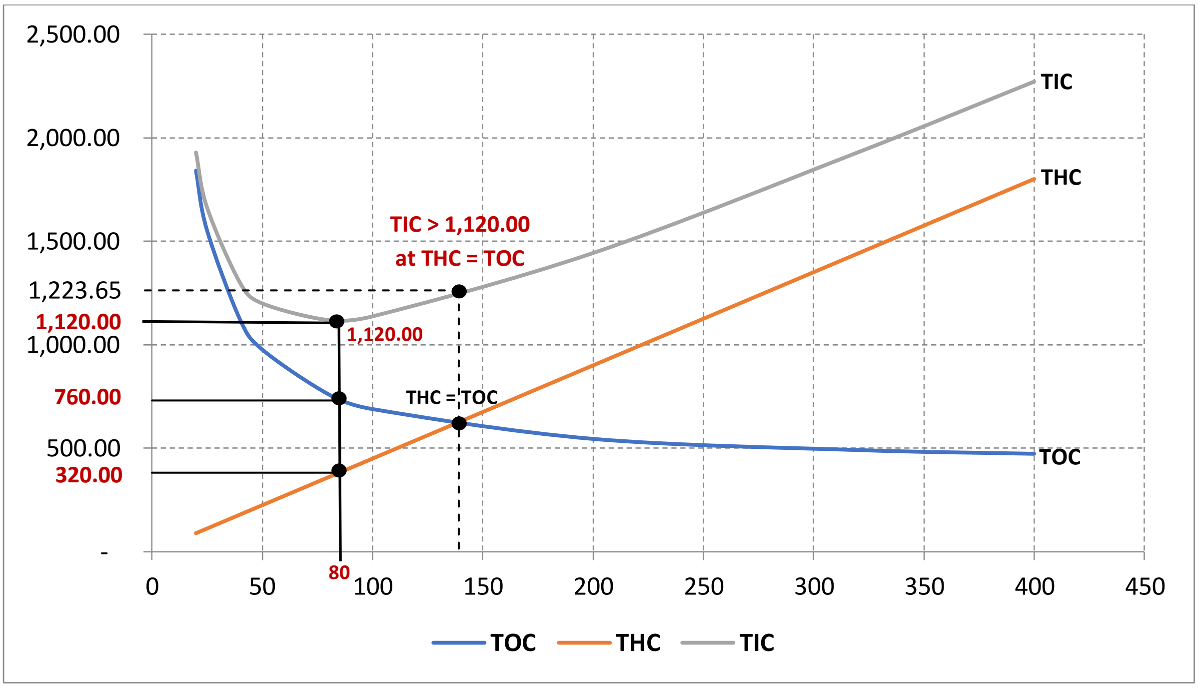
***N* = 11.7681** can result in $\boldsymbol{TOC = 400 + 18N = 611.8252, THC = \frac{7200}{N} = 611.8252}$ and

TIC = TOC + THC = 611.8252 + 611.8252 **= 1223.65** and note that ***this is not the minimum***.

Table 3: TOC, THC and TOC with respect to the quantity to be ordered per order

Ordering Quantity (Q)	Number of Orders (N)	Total Ordering Cost (TOC) $TOC = 400 + 18N$	Total Handling Cost (THC) $THC = \left(\frac{Q}{2}\right) * C_H$	TIC = TOC + THC
20	80	1,840.00	90.00	1,930.00
25	64	1,552.00	112.50	1,664.50
40	40	1,120.00	180.00	1,300.00
50	32	976.00	225.00	1,201.00
80	20	760.00	360.00	1,120.00
100	16	688.00	450.00	1,138.00
200	8	544.00	900.00	1,444.00
320	5	490.00	1,440.00	1,930.00
400	4	472.00	1,800.00	2,272.00

Figure 2: TOC, THC and TOC with respect to the quantity to be ordered per order



Where TOC = Total Ordering Cost, THC = Total Handling Cost, TIC = Total Incremental Cost = **THC + TOC**, and X-axis refers to the quantity of the material (Q) to be placed in an order.

It is also notable that TOC and THC becomes equal (***TOC = THC***), when about the ordering quantity is 135.9612 per order that can again result in ***TOC = THC*** = 611.8252 and ***TIC*** = 1,223.65. The above results can be obtained by equating the both ***TOC*** and ***THC*** and solving for ***Q***, where consider

$$TOC = 400 + 18\left(\frac{1600}{Q}\right) \quad \text{and} \quad THC = \left(\frac{Q}{2}\right) * 9, \quad \text{then equate } TOC \text{ and } THC \text{ to solve for } Q.$$

Now, it is possible to confirm the following.

Annual Demand (***D***) = 1600 unit of tons,

Daily Usage (***d***) = 8,

Annual working days (***AWD***) = 200

Handling Cost per unit (C_H) = 9

Annual Fixed Cost of orders = 400

Variable cost per order = 18

Single order cost (C_O) = 400 + 18***N***

Economic Number of Orders (***ENO***) = 20

Economic Order Quantity (***EOQ***) = 80

Minimised Total Incremental Cost (***TIC***) = 1,120.

Considering the above calculations, the cycle time (***T****) of an economic order/quantities can be determined in two way.

$$\text{Cycle Time } \boldsymbol{T^*} = \frac{\boldsymbol{EOQ}}{\boldsymbol{Daily\ Usage}} = \frac{\boldsymbol{Q^*}}{\boldsymbol{d}} = \frac{\mathbf{80}}{\mathbf{8}} = \mathbf{10} \text{ days} \qquad \text{OR}$$

$$\text{Cycle Time } \boldsymbol{T^*} = \frac{\boldsymbol{Number\ of\ annual\ working\ days}}{\boldsymbol{Economic\ Number\ of\ Orders}} = \frac{\boldsymbol{AWD}}{\boldsymbol{ENO}} = \frac{\mathbf{200}}{\mathbf{20}} = \mathbf{10} \text{ days}$$

Exhibit 2: Unit Handling Cost varies with the ordering size of a product

In this exhibit, the assumption "***constant handling cost per unit of an item***" of the basic EOQ model has been relaxed with the size of the ordering quantity in an order. In this context, the following information can be considered to understand the relaxed assumption of constant unit handling cost.

RAM Ltd. has demand for its material ***D =*** 48,600 units of tons that accounts for 270 working days (***AWD*** = 270) at the usage rate of 180 units of tons per day (***d*** = 180). In this context, the unit handling cost (C_H) varies with the ordering size of the material that is usually accountable at the rate 2% of the ordering quantity (i.e., $C_H = 0.02Q$). However, there is a constant cost of $ 300.00 to place an order (i.e., C_O = 300).

In summary, ***D =*** 48,600 ***AWD*** = 270 ***d*** = 180 $C_H = 0.02Q$ and C_O = 300.

Accordingly,

$$TOC = \left(\frac{D}{Q}\right) * C_O = \left(\frac{48600}{Q}\right) * 300 = \frac{14{,}580{,}000}{Q} \qquad \text{and}$$

$$THC = \left(\frac{Q}{2}\right) * C_H = \left(\frac{Q}{2}\right) * 0.02Q = 0.01\ Q^2$$

Hence, the Total Incremental Cost (TIC) can be given as

$$TIC = TOC + THC = \frac{14{,}580{,}000}{Q} + 0.01\ Q^2$$

Differentiating TIC with respect to Q can result in

$$\frac{d(TIC)}{dQ} = \frac{-14{,}580{,}000}{Q^2} + 0.02\ Q \qquad \text{and for maximum or minimum } \frac{d(TIC)}{dQ} = 0.$$

Therefore, $\frac{-14{,}580{,}000}{Q^2} + 0.02\,Q = 0$ and $14{,}580{,}000 = 0.02\,Q^3$

$$Q^3 = \frac{1{,}458{,}000{,}000}{2} = 729{,}000{,}000 \quad \text{and} \quad \boldsymbol{Q = 900}$$

If ***Q*** = 900 gives the minimum of ***TIC***, the second derivative of ***TIC*** (= ***TOC + THC***) should give ***greater than zero*** (> 0) for ***Q*** = 900.

Accordingly,

$$\frac{d(TIC)}{dQ} = \frac{-14{,}580{,}000}{Q^2} + 0.02\,Q \quad \text{and} \quad \frac{d^2(TIC)}{dQ^2} = \frac{29{,}160{,}000}{Q^3} + 0.02$$

Substituting Q = 900 in $\left[\frac{d^2(TIC)}{dQ^2}\right]$ can result in

$$\frac{d^2(TIC)}{dQ^2} = \frac{29{,}160{,}000}{900^3} + 0.02 = 0.04 + 0.02 = 0.06 \text{ and this gives } \frac{d^2(TIC)}{dQ^2} = 0.06 > 0$$

Hence, TIC gives minimum cost when Q = 900.

Thus, $\boldsymbol{TIC = TOC + THC = \frac{14,580,000}{900} + 0.01 * 900^2 = 16,200 + 8,100 = 24,300}$

Also, $TOC = \left(\frac{D}{Q}\right) * C_O = \left(\frac{48600}{900}\right) * 300 = \frac{14{,}580{,}000}{900} = \boldsymbol{16,200}$

and $THC = \left(\frac{Q}{2}\right) * C_H = \left(\frac{900}{2}\right) * 0.02 * 900 = 0.01 * 900^2 = 0.01 * 810{,}000 = \boldsymbol{8,100}$

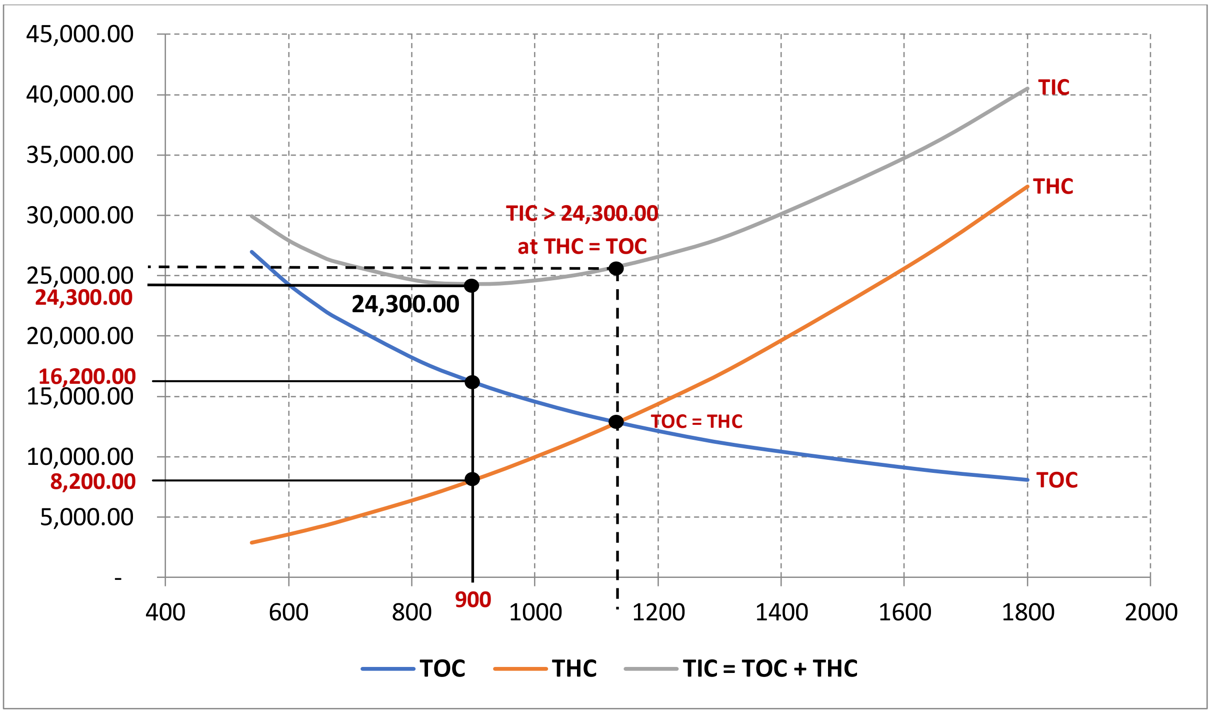
In this case also, it is notable that the EOQ has been derived at minimum of TIC when TOC ≠ THC (i.e., 16,200 ≠ 8,100).

All the above information can be shown in an illustration with a diagram (see Table 4 and Figure 3).

Table 4: TOC, THC and TOC with respect to the quantity to be ordered in an order

Ordering Quantity (Q)	**Number of Orders (N)**	**Total Ordering Cost (TOC)** $\boldsymbol{TOC = 400 + 18N}$	**Total Handling Cost (THC)** $\boldsymbol{THC = \left(\frac{Q}{2}\right) * C_H}$	**TIC = TOC + THC**
1800	27	8,100.00	32,400.00	40,500.00
1620	30	9,000.00	26,244.00	35,244.00
1350	36	10,800.00	18,225.00	29,025.00
1215	40	12,000.00	14,762.25	26,762.25
1080	45	13,500.00	11,664.00	25,164.00
972	50	15,000.00	9,447.84	24,447.84
900	54	16,200.00	8,100.00	24,300.00
810	60	18,000.00	6,561.00	24,561.00
675	72	21,600.00	4,556.25	26,156.25
648	75	22,500.00	4,199.04	26,699.04
600	81	24,300.00	3,600.00	27,900.00
540	90	27,000.00	2,916.00	29,916.00

Figure 3: TOC, THC and TOC with respect to the quantity to be ordered in an order



Now, it is possible to confirm the following.

Annual Demand (***D***) = 48,600 unit of tons,

Annual working days (***AWD***) = 270

Single order cost (C_O) = 300

Economic Number of Orders (***ENO***) = 54

Daily Usage (***d***) = 180

Handling Cost per unit (C_H) = 0.02 ***Q***

Economic Order Quantity (***EOQ***) = 900

Minimised Total Incremental Cost (***TIC***) = 24,300.

Considering the above calculations, the cycle time (***T****) of an economic order/quantities can be determined in two way.

$$\text{Cycle Time } \boldsymbol{T}^* = \frac{\boldsymbol{EOQ}}{\boldsymbol{Daily\ Usage}} = \frac{\boldsymbol{Q}^*}{\boldsymbol{d}} = \frac{\mathbf{900}}{\mathbf{180}} = \mathbf{5} \text{ days} \qquad \text{OR}$$

$$\text{Cycle Time } \boldsymbol{T}^* = \frac{\boldsymbol{Number\ of\ annual\ working\ days}}{\boldsymbol{Economi\ Number\ of\ Orders}} = \frac{\boldsymbol{AWD}}{\boldsymbol{ENO}} = \frac{\mathbf{270}}{\mathbf{54}} = \mathbf{5} \text{ days.}$$

3. CONCLUDING REMARKS

The primary EOQ model with many identical assumptions demonstrates that the EOQ can be determined where THC = TOC and the TIC (= TOC + THC) gets minimised. However, this paper aims to explain the determination of the EOQ under the condition: TOC ≠ THC. In this context, two assumptions: (a) the constant unit handling cost, and (b) the constant per order cost, are relaxed with their varying conditions. Accordingly, this paper has presented two exhibits as illustrative examples to make the reader understand the purpose of this paper.

Exhibit 1 considers relaxing the assumption of constant per order cost and illustrates with the concepts of fixed cost and variable cost for placing an order. This exhibit demonstrates a situation where the EOQ (or ENO) can be determined when THC ≠ TOC. Similarly, Exhibit 2 explains how the assumption of constant handling cost can be relaxed and illustrates with the varying cost of handling cost per unit with respect to the quantity (Q) to be placed in an order. Notably, for both illustrations of the exhibits appropriate tables and diagrams are also supportively presented for further and clear understanding of the paper.

References

Anderson, D.R., Sweeney, D.J., and William, T.A. (2006), Quantitative Methods for Business, 10th ed., India: Thomson South Western.

Hax, A.C. and Candea, D. (1984). ***Production and Operations Management***, Englewood Cliffs, NJ: Prentice-Hall (ISBN 9780137248803)

Hillier, F.S., and Hillier, M.S. (2005). Introduction to Management Science – A modeling and Case Studies Approach with Spreadsheets, 2nd ed., New Delhi: Tata McGraw-Hill Publishing Company Limited.

Kalavathy, S. (2002). Operations Research, 2nd ed., Delhi: Vikas Publishing House Pvt Ltd.

Karunaratna. K. R. M. T. (2005). Quantitative Methods for Management, 3rd ed., Sri Lanka.

Lawrence, J.A., and Pasternack, B.A. (2002). Applied Management Science, 2nd ed., India: John Wiley & Sons, Ins, India

Pinney, W.E., and McWilliams, D.B. (1987). Management Science: An Introduction to Quantitative, Analysis for Management, 2nd ed., New York: Harper and Row, Publishers.

Taha, H.A. (2004). Operations Research: An Introduction, 7th ed., New Delhi: Prentice-Hall of India Private Ltd.