ECONOMIC ORDER QUANTITY (EOQ)

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Abstract: In stock management, Economic Order Quantity (EOQ) is an important inventory management system that demonstrates the quantity of an item to reduce the total cost of both handling of inventory (Handling Cost) and order processing (Ordering Cost). The purpose of determining the EOQ is to minimise the Total Incremental Cost (TIC), beyond the cost of purchasing of a product/material, in consideration of two main total costs: Total Ordering Cost (TOC) and Total Handling Cost (THC). This paper contextually highlights two basic methods of determining the EOQ: Trial and error method and Mathematical approach and emphasises the mathematical model as highly useful to enhance the inventory management of a product.

- Keywords: economic order quantity, incremental cost, ordering cost, handling cost, lead-time, safety stock
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Abstract: In stock management, Economic Order Quantity (EOQ) is an important inventory management system that demonstrates the quantity of an item to reduce the total cost of both handling of inventory (Handling Cost) and order processing (Ordering Cost). The purpose of determining the EOQ is to minimise the Total Incremental Cost (TIC), beyond the cost of purchasing of a product/material, in consideration of two main total costs: Total Ordering Cost (TOC) and Total Handling Cost (THC). This paper contextually highlights two basic methods of determining the EOQ: Trial and error method and Mathematical approach and emphasises the mathematical model as highly useful to enhance the inventory management of a product.

1. INTRODUCTION

Various aspects are very important in warehouse management system, such as inventory management, warehouse maintenance, overhead management, pricing systems, etc. However, determining optimal ordering quantity is one of the main aspects in inventory management that can facilitate the inventory management to run with optimal cost. In this context, this paper is devised to illustrate the basic model of *Economic Order Quantity* (*EOQ*) from a learner's point of view.

The purpose of determining the EOQ is to minimise the Total Incremental Cost (TIC), beyond the cost of purchasing, in consideration of two main total costs: Total Ordering Cost (TOC) and Total Handling Cost (THC).¹ In this context, this paper highlights two basic methods of determining the EOQ: Trial and error method and Mathematical approach. However, in this illustration, mathematical model is highly emphasised to enhance the inventory management applications.

As further explanations, EOQ related other measures also illustrated supports to the inventory management system, mainly the relationships of EOQ to Economic Number of Orders (ENO), length of inventory cycle, and reorder point of quantity stored. As a contextual explanation of EOQ, this paper has been following with: Definition and determination of EOQ, Extensions of EOQ with other related concerns and Concluding remarks.

¹ An incremental cost is the difference in total costs as the result of a change in some activity. Incremental costs are also referred to as the differential costs and they may be the relevant costs (source: <u>https://www.accountingcoach.com/blog/what-is-an-incremental-cost</u>).

2. DEFINITION AND DETERMINATION OF ECONOMIC ORDER QUANTITY (EOQ)

Economic Order Quantity (EOQ) is an inventory management system that demonstrates the quantity of an item to reduce the total cost of both handling of inventory (Handling Cost) and order processing (Ordering Cost). EOQ as a model has been introduced in 1913 by Ford W. Harris; and R. H. Wilson and K. Andler are given credit for their in-depth analysis and application of the EOQ model (Hax and Candea, 1984).

With respect to an item to be ordered, from a business point of view, the EOQ model establishes the amount of quantity to be placed in an order in consideration of minimising the annual total cost of inventory handling and order processing. In this context, these specific two types of costs are the main categories of determining the EOQ in its basic explanation. However, the model has been presented with certain assumptions for the initial understanding; and from that point onward, its extensions are used widely in businesses, especially in inventory management.

2.1 Assumptions of the Basic Model of Economic Order Quantity (EOQ)

The determination of EOQ consists of the following assumptions:

- a) The EOQ will be determined for every product individually in a business.
- b) Annual requirement (Demand) for product in units is known with certainty.
- c) Ordering cost is known and constant throughout the year.
- d) Inventory handling cost is known and constant throughout the year. Notably, if the handling cost of an item is given as the percentage of price of the item, the unit price of the item remains same throughout the year.
- e) No cash or quantity discount is allowed.
- f) The ordered quantity of the product is delivered at once as a single batch.
- g) Immediate replenishment of ordered quantity on time (No delay and stock shortage).
- h) Constant lead time is only allowed (no fluctuation is permitted).

2.2 Annual Demand of the Product

The annual requirement of a product is constant and known in its measurement units and this known as annual demand of the product (*D*). Using various forecasting technique, it is important for a business to predict the annual demand for the specific item, for which the business need to know the EOQ. As the demand produces the primary purchasing cost of the item, the total purchasing cost is irrelevant in determining the EOQ.

2.3 Order Processing Cost

The Ordering Cost refers to the cost of orders to be placed for the product in consideration of order communication, allowances to purchase officers, order printing and stationery, costs of inspection, receiving the product, and transport cost, etc. Notably, these costs remain constant and unchanged for

the period, irrespective the number of orders to be placed. As the ordering cost per order is constant, the relationship between the quantity ordered and number of orders to be placed is negative, i.e., higher the quantity ordered (Q) per order, lower the number of orders to be placed; and lower the quantity ordered (Q) per order, higher the number of orders to be placed. This implies the negative relationship between the quantity ordered (Q) and total cost of order processing (TOC) as in Figure 1.

2.4 Inventory Handling Cost

This cost refers to the handling and maintaining the product inventory in workplace in consideration of warehousing costs, shrinkage loss, evaporation, deterioration and spoilage costs, insurance, warehouse rent, obsolescence, and other related overhead cost of warehouse. As these costs are constant to maintain the total demand (annual requirement) of the product, the handling cost per unit remains same. Therefore, total cost of handing has positive relationship with the number of products handled in the workplace (warehouse), i.e., higher the number of products (Q) in store, higher the total cost of handling (THC); and lower the number of products in store, lower the total handling cost (THC) as in Figure 2. It is notable that handling of stock would be a half the number of quantity to be ordered throughout the year; and therefore, the average stock to handle would be Q/2.



2.5 The Model: Economic Order Quantity (EOQ)

The basic EOQ model, with all assumptions in consideration, deal with two types of costs: Total Ordering Cost (TOC) and Total Handling (THL). It is obvious from the above explanation that these costs are moving in opposite directions, when certain number of quantities are ordered for storage purpose. Therefore, it is important for a business to find a trade-off point of ordering quantity in order to minimise the total cost of both: TOC plus THC. In this context, the quantity to be ordered to minimise the total cost of both TOC and THC is known as the *Economic Order Quantity (EOQ)*.

As TOC has negative relationship to quantity to be ordered and THC has positive relationship to quantity to be ordered, the total minimum cost of both TOC and THC is the intersection point of both cost lines that can produce: (a) the total cost of both TOC and THC as minimum as possible; and (b) the number of quantities to be ordered (*known as EOQ*) to meet the minimised cost (see Figure 3).



Figure 3: Graphical Determination of Economic Order Quantity

Notably, from purchasing point of view, TOC and THC are the additional costs, which incur above cost of a material purchased. Therefore, the aggregation of both costs (TOC and THC) are known as Total Incremental Cost, i.e., *TIC = TOC + THC*. In the context of EOQ, TOC and THC are the additional costs incurring beyond the original purchasing cost of an item.²

Generally, there are two basic methods to determine the EOQ.: (a) Trial and Error method in combination of graphical representation; and (b) Mathematical Approach – this is widely used popular method. These methods can be explained with an exhibit for easy understanding.

Exhibit 1

Consider a small production process, which need sawdust as raw material. The production process requires 20,000 cubic meters (m³) annually. If an order is to be placed, every order can cost \$ 50.00 and the cost of handling one unit of cubic meter is \$ 2.

From the above, the following information is available.

Annual Demand (D) = 20000 m^3 for the sawdust,

² An incremental cost is the difference in total costs as the result of a change in some activity. Incremental costs are also referred to as the differential costs and they may be the relevant costs (source: <u>https://www.accountingcoach.com/blog/what-is-an-incremental-cost</u>).

Cost per order (C_0) = \$50.00 and Handling cost per unit (C_H) = \$2.00

Following table can produce various possible number of cubic meters of sawdust to be ordered (this should be assumed independently) and the related cost calculations.³

Ordering Quantity (Q)	No. of Orders (N = D/Q)	Average Stock to handle (Q/2)	Total Ordering Cost (TOC) @ \$50	Total Handling Cost (THC) @ \$2	Total Incremental Cost (TIC = TOC + THC)
250	80	125	4,000.00	250.00	4,250.00
400	50	200	2,500.00	400.00	2,900.00
500	40	250	2,000.00	500.00	2,500.00
1000	20	500	1,000.00	1,000.00	2,000.00
2000	10	1000	500.00	2,000.00	2,500.00
2500	8	1250	400.00	2,500.00	2,900.00
4000	5	2000	250.00	4,000.00	4,250.00

Table 1: Various number of cubic meters (quantity) and related cost calculations

From the table, it is possible to observe that the (last) column TIC has a minimum of \$2000.00, where TOC = THC =\$1000.00 and the quantity Q = 1000. The information available in the table can be shown in a diagram with all three costs: TIC, TOC and THC (see Figure 4).





³ As various quantities are assumed independently for table formulation, this method is called trial and error method.

Using the same information of Exhibit 1, the following deals with the mathematical approach, which is widely used in determining the EOQ in inventory control system.

The basics of mathematical model therefore need to be illustrated to determine the EOQ to have minimum of TIC. Accordingly, it is important to determine total individual cost of both (TOC and THC), in terms of quantities to be ordered and handled.

Therefore, $TOC = Cost per order \cdot (Demand/Quantity Ordered per year)$ $TOC = C_0 \cdot (D/Q) = \frac{DC_0}{Q}$ and THC = Cost per unit for handling . (Average quantity maintained in store for a year) THC = C_H · (Q/2) = $\frac{QC_H}{2}$

This results in TIC = TOC + THC = $\frac{DC_O}{Q} + \frac{QC_H}{2}$

As this function TIC depends on the quantity ordered (Q) to minimise the total cost, the function needs to be differentiated with respect to Q.

This can result in:

 $\frac{d(TIC)}{dQ} = \frac{-DC_0}{Q^2} + \frac{C_H}{2} \quad \text{and for minimisation/maximisation } \frac{d(TIC)}{dQ} = 0.$ Therefore, $0 = \frac{-DC_0}{Q^2} + \frac{C_H}{2} \quad \text{and} \quad \frac{C_H}{2} = \frac{DC_0}{Q^2}; \quad \text{and solving for Q can result in}$ $Q = \sqrt{\frac{2DC_0}{C_H}} \quad \text{and this must be confirmed for minimising the TIC.}$

To confirm the minimisation of TIC for a value of Q, the second derivative of TIC should be greater than zero for the value of Q.

As such, from the first derivative, the second derivative of TIC *must* result in

$$\frac{d^2(TIC)}{dQ^2} = \frac{2DC_0}{Q^3} \text{ and for a value of Q, } \frac{d^2(TIC)}{dQ^2} > 0.$$

Therefore, TIC optimally produce a minimum cost for the value of Q^* (where $Q^* = EOQ$).

Now, we can substitute the values available in Exhibit 1, where Annual Demand (D) = 20000 m³ for the sawdust, Cost per order (C_0) = \$ 50.00 and Handling cost per unit (C_H) = \$ 2.00.

Therefore, $TOC = \frac{DC_O}{Q} = \frac{(20000) \cdot (50)}{Q}$ and $THC = \frac{QC_H}{2} = \frac{Q \cdot (2)}{2} = Q$ Then, $TIC = \frac{(20000) \cdot (50)}{Q} + Q$ Differentiating TIC with respect to Q can result in $\frac{d(TIC)}{dQ} = \frac{(-20000) \cdot (50)}{Q^2} + 1$ For an optimum of TIC with respect to Q, $\frac{d(TIC)}{dQ} = 0$.

i.e., $0 = \frac{(-20000) \cdot (50)}{Q^2} + 1$ and solving for Q can result in $Q^2 = (20000) \cdot (50)$ and $Q^* = \sqrt{(20000) \cdot (50)} = 1000 \text{ m}^3.$

It is now possible to note that the number of orders to be place for the year is $(D/Q^*) = (20000/1000) = 20$, to have the optimal TIC.

 $Q^* = EOQ = 1000$ can be applied to determine the TOC, THC and TIC.

And

$$TOC = \frac{(20000) \cdot (50)}{(1000)} = \$ \ 1000, \quad THC = \frac{Q \cdot (2)}{2} = \frac{(1000) \cdot (2)}{2} = \$ \ 1000$$
$$TIC = TOC + THC = \$(1000 + 1000) = \$ \ 2000.$$

Explicitly, the results are obvious about the trade off point of Q (as EOQ) between the costs of TOC and THC as equal; and the TIC results in at the minimum of the sum of costs (TOC + THC).

3. EXTENSION OF EOQ WITH OTHER RELATED CONCERNS

As EOQ is explained to determine in both: Trial and Error Method and Mathematical Approach, it is notable how the EOQ determination can be useful to determine other related concerns. In this context, this section particularly deals with the following.

3.1 Determining TIC with Demand (D), per Order Cost (C₀) and per unit product Handling Cost (C_H)

As TIC = TOC + THC =
$$\frac{DC_O}{Q}$$
 + $\frac{QC_H}{2}$ and EOQ = Q* = $\sqrt{\frac{2DC_O}{C_H}}$, now Quantity (Q) in the TIC function

can be represented with EOQ = Q^{*}. Therefore, Q can be substituted with $\sqrt{\frac{2DC_O}{C_H}}$. This can result in

$$TIC = \frac{DC_{O}}{Q^{*}} + \frac{Q^{*}C_{H}}{2} = \left(\frac{DC_{O}}{\sqrt{\frac{2DC_{O}}{C_{H}}}} + \frac{\sqrt{\frac{2DC_{O}}{C_{H}}}}{2}\right) = \left(\frac{DC_{O}}{\sqrt{\frac{2DC_{O}}{C_{H}}}} + \frac{\sqrt{\frac{2DC_{O}C_{H}^{2}}{C_{H}}}}{2}\right) = \left(\sqrt{\frac{D^{2}C_{O}^{2}C_{H}}{2DC_{O}}} + \sqrt{\frac{2DC_{O}C_{H}^{2}}{4DC_{H}}}\right)$$
$$TIC = \left(\sqrt{\frac{DC_{O}C_{H}}{2}} + \sqrt{\frac{DC_{O}C_{H}}{2}}\right) = \left(2 * \sqrt{\frac{DC_{O}C_{H}}{2}}\right) = \sqrt{\frac{4*DC_{O}C_{H}}{2}}$$
$$TIC = \sqrt{2.D.C_{O}.C_{H}}$$

As in Exhibit 1, Annual Demand (D) = 20000 m³ for the sawdust, Cost per order (C₀) = 50.00 and Handling cost per unit (C_H) = 2.00,

$$TIC = \sqrt{(2) \cdot (20000) \cdot (50) \cdot (2)} = \sqrt{4,000,000} = \$ 2000.$$

3.2 Transforming EOQ into Economic (Optimum) Number of Orders (ENO)

When EOQ (Q*) is determined and termed as optimal quantity to minimise the TIC, the number of orders for the year can be determined as Demand (D) divided by EOQ (i.e., $N = D/Q^*$). This in other term can be interpreted that the Economic (optimal) Number of Orders (ENO = N*) can provide the minimum TIC. In this context, Q can be substituted in terms of N and the mathematical approach can be extended to determine the Economic Number of Orders (ENO) and TIC relatively, as shown below.

Therefore, TOC = Cost per order . (Demand/Quantity Ordered per year) $TOC = C_0 . (D/Q) = \frac{DC_0}{Q}$ and as N = (D/Q), $TOC = N . C_0$ THC = Cost per unit for handling . (Average quantity maintained in store for a year) $THC = C_H . (Q/2) = \frac{QC_H}{2}$ and as N = (D/Q), $THC = C_H . (Q/2) = \frac{DC_H}{2N}$

This results in TIC = TOC + THC = $\left(N.C_0 + \frac{DC_H}{2N}\right)$

As this function TIC depends on the quantity ordered (Q) to minimise the total cost, the function needs to be differentiated with respect to N.

This can result in:

Therefore,

$$\frac{d(TIC)}{dN} = C_0 + \left(\frac{-DC_H}{2N^2}\right) \text{ and for minimisation/maximisation } \frac{d(TIC)}{dQ} = 0.$$

$$0 = C_0 + \left(\frac{-DC_H}{2N^2}\right) \text{ and } C_0 = \frac{DC_H}{2N^2}; \text{ and solving for Q can result in}$$

$$N = \sqrt{\frac{DC_H}{2C_0}} \text{ and this must be confirmed for minimising the TIC.}$$

To confirm the minimisation of TIC for a value of N, the second derivative of TIC should be greater than zero for the value of N.

As such, from the first derivative, the second derivative of TIC *must* result in

$$\frac{d^2(TIC)}{dN^2} = \frac{DC_H}{N^3} \quad \text{and for a value of } \mathbf{Q}, \ \frac{d^2(TIC)}{dN^2} > 0.$$

Therefore, TIC optimally produce a minimum cost for the value of N* (where N* = ENO).

As in Exhibit 1, Annual Demand (D) = 20000 m³ for the sawdust, Cost per order (C₀) = 50.00 and Handling cost per unit (C_H) = 2.00, and substituting these values in

N* =
$$\sqrt{\frac{DC_H}{2C_O}} = \sqrt{\frac{(20000) \cdot (2)}{(2) \cdot (50)}} = 20.$$

With respect to the number of orders to be placed, ENO can be found can be represented in a diagram with TOC, THC and TIC, respectively (see Table 2 and Figure 5).

No. of Orders (N)	Ordering Quantity (Q = D/N)	Average Stock to handle (Q/2)	Total Ordering Cost (TOC) @ \$50	Total Handling Cost (THC) @ \$2	Total Incremental Cost (TIC = TOC + THC)
80	250	125	4,000.00	250.00	4,250.00
50	400	200	2,500.00	400.00	2,900.00
40	500	250	2,000.00	500.00	2,500.00
20	1000	500	1,000.00	1,000.00	2,000.00
10	2000	1000	500.00	2,000.00	2,500.00
8	2500	1250	400.00	2,500.00	2,900.00
5	4000	2000	250.00	4,000.00	4,250.00

Table 1: Various number of orders and related cost calculations

Figure 5: Economic Number of Orders (ENO) with TOC, THC and TIC



It is now possible to note that the number of quantities (known as EOQ in other term) to be placed in an order is $(D/N^*) = (20000/20) = 1000$, to have the optimal TIC.

3.3 Length of inventory cycle (provided with daily usage 'd' of the product)

Length of inventory cycle is a measure that gives a time period how long a batch of EOQ can last in the storage. As production/supply of the items takes place, the daily usage (d) of the item becomes

reduction in the EOQ stored. Therefore, length of inventory cycle indicated how many times (T) of daily usage of the item in total equates the EOQ.

where T =length of inventory cycle and d =daily usage of the item. Therefore, EOO = **T**.**d** $T = \left(\frac{EOQ}{d}\right) = \left(\frac{Q^*}{d}\right)$

Note that the daily usage should have been provided with certainty (consider it as an assumption).

Referring to the information above, EOQ = 1000 units and assume daily use (d) of the item is 100 units. Therefore, length of inventory cycle $T = \left(\frac{EOQ}{d}\right) = \left(\frac{1000}{100}\right) = 10$ days.

3.4 Determining number of annual working days

It is also notable that every batch of EOQ last for certain period (10 days in the example above) and this will happen for every order of EOQ in a year. Therefore, the T (=10 days) time cycle is applicable for every order. As EOQ can be transformed into Economic Number of Orders (ENO = N*) in a year and every order (a batch of EOQ) last for T (=10 days) time as a cycle, the annual number of working days in a year on this product can be determined as:

Annual number of working days = N^* . T^* = (20). (10) = 200 days

Note in the example, $N^* = (D/EOQ) = (20000/1000) = 20$ and $T^* = (EOO/d) = (1000/100) = 10.$

3.5 Reorder point quantity (provided with lead-time for stock replenishment)

The reorder point quantity/stock level of an item is measure at which the product needs an order placement for the replenishment of the stock, as for not to interrupt the trade operations. In other term, it is the stock level to use during the lead-time of stock replenishment. After the immediate replenishment of EOQ, daily usage of the product is taken from the stock in the store. In this context, reorder point can be determined in consideration of daily usage (d), lead-time of replenishing the EOQ (T_L) and safety stock (G_S) of the product/item.

• If a firm has no policy of maintaining a safety stock level of the item,

Reorder Level (ROL) = (Daily Usage). (Lead-time in days) R

$$ROL = (\boldsymbol{d}).(\boldsymbol{T}_L)$$

As previously stated, consider again the daily usage d = 100 units and the lead time $T_L = 6$ days. Accordingly, the reorder level ROL = (d). $(T_L) = (100)$.(6) = 600 units. This implies that when the stock level is at 600 units a new order of EOQ need to be placed to get it after 6 days as immediate replenishment, and available 600 units can meet the 6 days requirements at the daily usage of 100 units (see Figure 6).



Figure 6: Daily stock level and Reorder Level (ROL) without safety stock

• If a firm maintains a safety stock level of the item, Reorder Level (ROL) = (Daily Usage) . (Lead-time in days) + (Safety Stock) ROL = $(d).(T_L) + (G_S)$

When we consider a safety stock level $G_S = 200$ units of the product as an additional information, the reorder level ROL = $(d).(T_L) + (G_S) = (100).(6) + 200 = 800$ units. This is to safeguard the daily operations of the firm in case of expected delay of two (2) days for replenishing the stock ordered (EOQ). This implies that a deviation of additional two days to the lead time cannot have impacts on continuing operations of the firm (see Figure 7).





12 http://ssrn.com/abstract=3475239 Further, there is a possibility of having irregular daily usage of the product/item. In this case, it is wise to have the maximum daily usage of the product to determine its ROL. This always happen when there is uncertainty of daily usage with varying demands, considering maximum usage of the item can facilitate the continuing operations. In case of '*No Safety Stock*', ROL = (Maximum of *d*).(*T*_L); and if '*With Safety Stock*', ROL = (Maximum of *d*).(*T*_L) + (*G*_S).

4. CONCLUDING REMARKS

In inventory management, determination of EOQ is an important measure to regulate other concerns in warehouse management. Main objective of determining the EOQ is to minimise the total incremental cost (TOC and THC) that incur beyond the cost of purchasing the product. In this context, this paper attempts to highlight two basic methods of determining the EOQ: Trial and Error Method and Mathematical Approach. However, it is advisable to apply mathematical approach to make decisions objectively.

This paper has more explanations on mathematical approach of EOQ and further explanations are also provided with the relationships of EOQ to Economic Number of Orders (ENO), length of inventory cycle, and reorder point of quantity stored. This paper is contextually presented for the learners of EOQ within its assumptions. However, the assumptions themselves become the limitations of the model. This explanation can be extended in particular by analysing: (a) How an EOQ measure can be determined where TOC \neq THC, (b) What will happen, if per order cost and/or per unit handling cost is dependent on EOQ/ENO, (c) What will happen to EOQ, if discount is allowed, and (d) Sensitivity Analysis, relatively. Considering them, this paper can provide a base those extensions.

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